Laminar Dispersion of Polymer Solutions in Helical Coils

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Synopsis

In the present study the step response experiments were carried out with power law fluids in two helical coils to examine the suitability of axial dispersed plug flow model in describing the laminar dispersion of non-Newtonian fluids in helical coils. The ranges of variables covered are $10 \leq \lambda \leq 100$, $0.01 \leq N_{\mathrm{Regen}} \leq 2.5$, $0.001 \leq N_{\mathrm{De}} \leq 0.77$ and $0.035 \leq \tau \leq 1.33$. It is found that coiling results in reduced dispersion to that in a straight tube.

INTRODUCTION

The effect of curvature on flow patterns is to induce secondary flow; this in turn leads to an increased pressure drop, higher heat and mass transfer coefficients, and reduced spread of residence times in comparison to the flow (at the same Reynolds number) through a straight tube. The helical coils thus find extensive use in industrial practice. Flow of non-Newtonian fluids is of considerable pragmetic interest and this flow in coiled tubes is of a particular interest in a variety of fields. Its possible applications are determination of residence time of tracer solutes injected into the bloodstream, the transport of slurries and melts through curved tubes, the design of the flow reactors for biological systems.

Since the pioneering work of Taylor[1,2] numerous articles have discussed dispersion in variety of situations. The prior literature on dispersion in the laminar flow of Newtonian and non-Newtonian fluids through straight and coiled tubes has been reviewed by Singh and Nigam.[3] However, most articles have been confined to disperson in Newtonian fluids. Very little work is reported on dispersion in laminar flow of non-Newtonian fluids through straight tubes[4–6] and no work is reported on helical coils. From the point of view of reactor performance and the importance of non-Newtonian fluids in industries, it was therefore decided to study the dispersion of polymer solutions through helical coils.

THEORY

In the present study the dispersion model solution for doubly infinite boundary conditions[7] was used. The solution can be written

$$F = \frac{1}{2}\left\{1 - \operatorname{erf}\left[\frac{1-\theta}{2}\left(\frac{\bar{u}L}{D}\frac{1}{\theta}\right)^{1/2}\right]\right\} \tag{1}$$

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where F is the dimensionless outlet concentration, $D/\bar{u}L$ is dispersion number, and θ is dimensionless time.

The values of the Peelet number were computed by the method of matching the experimental and theoretical curves. The method is based on the nonlinear least-squares-fit technique.[8] The criterion for the validity of dispersion model was that if more than 1.25% of the fluid was assigned on incorrect residence time by the model, it was rejected.[9]

EXPERIMENTAL

Step response experiments were carried out with polymer solutions using Congo Red Dye as tracer material in two helical coils to examine the suitability of the axially dispersed plug flow model in describing the laminar dispersion of polymer solutions in helical coils. The helical coils having curvature ratios (λ) of 10.48 and 101.2 were prepared by winding a thick walled, flexible PVC tube of 0.3 cm i.d. around smooth pipes. The length of the PVC tube in each case was kept at 1509 cm, so that the effect of developing a velocity profile on residence time distribution was minimized.

Aqueous polymer solutions of 1 and 2% by weight were prepared with low-viscosity-grade carboxy methyl cellulose (CMC) supplied by BDH (England). The rheolological data were obtained by a capillary viscometer. The molecular diffusion coefficient of Congo Red Dye (tracer) in CMC solutions, together with the rheological data, were reported by Singh and Nigam.[10]

The procedure for determining the residence time distribution is described in the authors' previous paper.[10] In the present study a tracer solution of 0.03 g/liter was used in all experiments.

The ranges of variables covered are $10.40 < \lambda < 101.2$, $0.0097 < N_{\mathrm{Regen}} < 2.51$, $0.001 < N_{\mathrm{De}} < 0.775$, $13.4 < \bar{t} < 498$ min, and $0.035 < \tau < 1.327$, where N_{Regen} denotes generalized Reynolds number and N_{De} denotes Dean's number. These values were thought to be fairly representative of those likely to be met in industrial practice.

RESULTS AND DISCUSSION

The step response experiments were carried out with aqueous CMC solution of 1 and 2% by weight in both coils. The general nature of the response curve was similar to that reported for Newtonian fluids.[9] However, for polymer solutions, no oscillations in the F curve were observed. Figure 1 shows typical F curves for coil of curvature ratio (λ) of 10.48 with 2% aqueous CMC solution. In the case of a 1% aqueous CMC solution, the RTD (Residence Time Distribution) curves were less abrupt. Some experiments were also conducted to study the effect of CMC concentration on laminar dispersion in helical coils. The value of characteristic time (τ) was kept the same in both cases. It should be noted that in order to keep the same τ, a higher mean time was required in the case of the 2% CMC solution, because the molecular diffusivity of Congo Red Dye in a 2% CMC solution was about one-third of that in the 1% CMC solution. This may be the reason why the 2% CMC solution gives an RTD broader than that of the 1% CMC solution at the same value of τ.

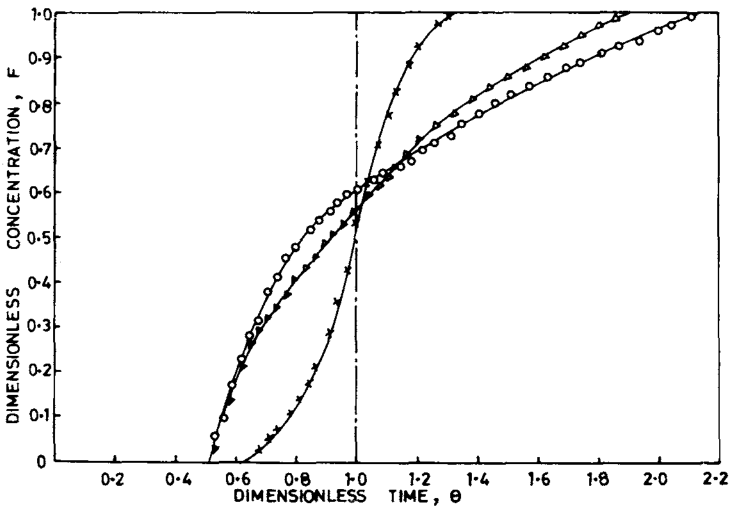

Fig. 1. Step input response for helical coil. $\lambda = 10.48$ and 2% CMC solution. $\tau = 0.036$ (○-○-○), 0.10 (△-△-△), 0.30 (×-×-×).

Effect of Generalized Reynolds Number

The dispersion numbers were calculated using the method of matching experimental and theoretical F curves. The generalized Reynolds numbers for power law fluids were calculated as

$$N_{\text{Regen}} = \left(\frac{4n}{3n+1}\right)^n \frac{1}{8^{n-1}} \left(\frac{d_t^n \,\bar{u}^{2-n}\rho}{K}\right) \tag{2}$$

The values of the dispersion number and the generalized Reynolds number were plotted in Figure 2, only for those cases where the dispersion model was found to be applicable. However, with the limited number of data it could be argued that unlike the Newtonian fluid results of Trivedi and Vasudeva,[9] a smooth relationship exists with the dispersion number and the generalized Reynolds number. A tenfold reduction in curvature ratio (λ) results in a twofold reduction in the value of the dispersion number at the same generalized Reynolds number.

Reduction in Coils over Straight Tube

Erdogan and Chatwin,[11] using Dean's[12] velocity profile and Nunge et al.[13] using the Topakoglu[14] velocity profile for coils, derived a theoretical expression for dispersion in coils for Newtonian fluids under the conditions of appreciable influence of molecular diffusion. They have shown that $D\, D_m/\bar{u}^2\, d_t^2 = K_c$ is a measure of dispersion in coils. Trivedi and Vasudeva[9] have correlated their Newtonian fluid results in terms of K_c versus the Dean number over a wide range of system parameters. In the present study the value of K_c was calculated for cases where the dispersion model was applicable to coils. Using the Fan and Hwang[4] analysis for power law fluids, the values of K_c were also calculated for straight tubes of the same dimensions. Under the conditions of applicability of the dispersion model, the reduction in the axial dispersion owing to coiling in laminar flow of power law fluids can be of the order of 1.6-fold to fivefold depending upon the system parameter.

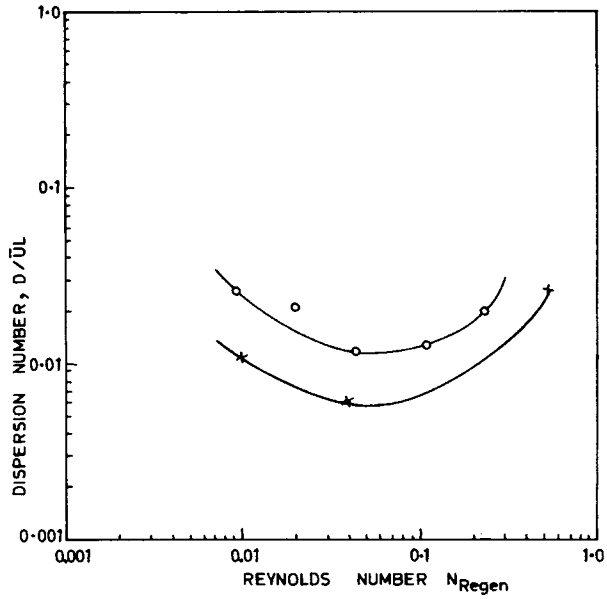

Fig. 2. Dispersion number for laminar flow in helical coil: λ, 10.48 (×); λ, 101.20 (o).

Conditions for the Validity of the Dispersion Model

The essence of the theory of dispersion is to establish the conditions under which dispersion model holds good. Gill et al.[15] have shown that for a straight tube the dispersion model holds for Newtonian fluids when $\tau > 0.5$, and Shah and Cox[5] have shown that the model holds for power law fluids when $\tau > 0.7$. Nunge et al.[13] developed a criterion for minimum τ above which the dispersion model theory may be used for curved tubes. The criterion may be written

$$\tau_{\min} \geqslant 101\, K_c^{0.9} \tag{3}$$

Their relationship is valid in the case of Newtonian fluids for very low values of a Reynolds number. More recently, Trivedi and Vasudeva,[9] from their experimental results for Newtonian fluids in the case of coils, derived the following relation for the applicability of dispersion model:

$$\tau_{\min} > 6\, N_{\mathrm{Re}}^{-1} \tag{4}$$

The experimental results of the present study were examined to obtain the approximate value of $\tau_{\min}$ above which the dispersion model may be expected to hold. The value of τ used in the present study in the case of 1 and 2% aqueous CMC solutions are plotted against a generalized Reynolds number in Figure 3. Two parallel lines emerge for the two different aqueous CMC solutions. A demarcation between the solid circles (representing cases where the dispersion model hold) and open circles (representing cases of inapplicability of the dispersion model) suggests the values of $\tau_{\min}$ above which the dispersion model can be accepted to hold. The condition for validity of the dispersion model in power law fluids over the ranges studied in the present work may be stated as

$$\tau_{\min} > 0.12\, N_{\mathrm{Regen}}^{-0.27} \tag{5}$$

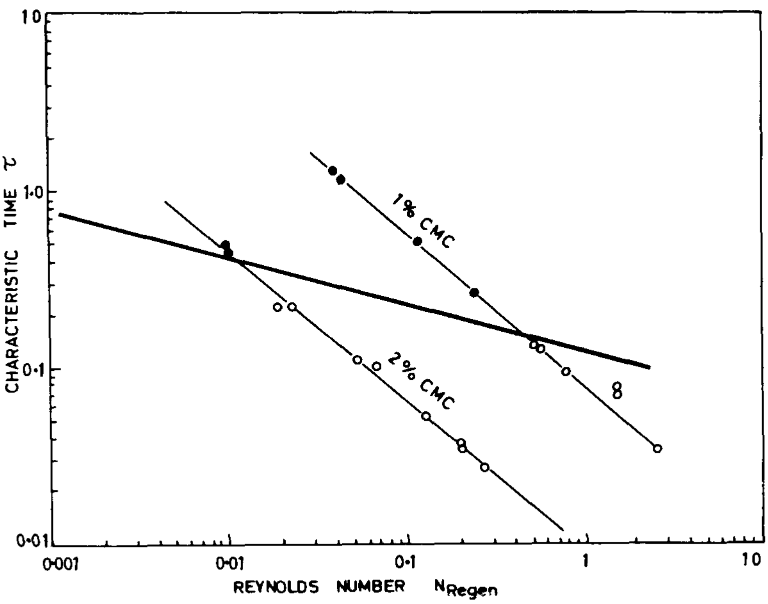

Fig. 3. Region of applicability of dispersion model.

The values of τ required for the validity of the dispersion model in the case of the coils were less than those in straight tubes.

CONCLUSIONS

The suitability of an axially dispersed plug flow model to represent the laminar dispersion of polymer solutions in helical coils over a wide range of conditions has been examined. An approximately fivefold reduction in axial dispersion was obtained under the experimental conditions by coiling the tubes for aqueous polymer solutions.

NOMENCLATURE

C_m	bulk mean concentration at the outlet, g/liter
C_0	tracer concentration at the inlet, g/liter
d_c	coil diameter, cm
d_t	tube diameter, cm
D	effective diffusion coefficient, cm^2/sec
D_m	molecular diffusion coefficient, cm^2/sec
F	dimensionless concentration at the outlet ($= C_m/C_0$)
K	power law consistency index, g sec^{n-2}/cm
K_c	dimensionless number ($= DD_m/\bar{u}^2 d_t^2$)
L	tube length, cm
n	flow index
N_{De}	Dean number ($= N_{Regen}/\sqrt{\lambda}$)
N_{Regen}	generalized Reynolds number defined by eq. (2)
t	time, sec
$\bar{t}$	mean holding time, sec
$\bar{u}$	average linear velocity of fluid cm/sec
λ	coil to the tube diameter ratio ($= d_c/d_t$)

ρ	fluid density, g/ml
θ	dimensionless time ($= t/\bar{t}$)
τ	dimensionless characteristic time ($= 4\bar{t}\ D_m/d_t^2$)
$\tau_{\min}$	minimum value of τ above which dispersion model may hold, dimensionless

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